Physics 302 Photonics

HW-8 Chapter 10 SOLUTIONS

(10.7) This is a far field problem if $R >> D^2/\lambda$

But $D^2/\lambda = (0.1\,mm)^2/461.9\,nm = 0.02\,m$.

(a) Since $R = 1.0\,m >> 0.02\,m$, this is a far field problem

(b) Angular width of central maximum:

The $\frac{1}{2}$ width occurs when $\beta = \pi$

ie $\frac{kD}{2} \sin\theta = \pi \Rightarrow \sin\theta = \frac{2\pi}{kD} = \frac{\lambda}{D}$

ie $\sin\Theta = \left(\frac{461.9\,nm}{0.1\,mm}\right) \doteq 4.62 \times 10^{-3}\,rad \cong \theta$

$$\boxed{\text{Angular width} = 2\theta = 9.24 \times 10^{-3}\,rad = 0.529\,deg}$$

Since $\Theta \cong \frac{y}{L}$ $\Rightarrow$ $\boxed{\text{Width} = 2y = 2L\theta = 9.24\,mm}$

(10.8) Single slit w/ $m=0$ (dark fringe)

a) for destructive interference

$$D \sin\theta_m = m\lambda$$

$$D = m\lambda/\sin\theta_m = \frac{10\,(1152.2\,nm)}{\sin 6.2^\circ}$$

$$\boxed{D = 1.07\times 10^{-4}\,m = 0.107\,mm}$$

b) if immersed in water w/ $n=1.33$

$$\Rightarrow \sin\theta_m = \frac{m\lambda/n}{D} = \frac{10\,(1152.2\,nm)}{(1.07\times10^{-4}\,m)(1.33)} = 0.0810$$

$$\Rightarrow \boxed{\theta_{10} = 4.6^\circ}$$

(10.14) For Fraunhofer diffraction through any aperture, the electric field at point P (screen) is

$$E = \frac{\mathcal{E}_A e^{i(\omega t - kR)}}{R}\iint e^{ik(Yy+Zz)/R}\,dS \quad \text{where } dS = dy\,d$$

The irradiance is (up to a constant):

$$I(Y,Z) = \langle E^2\rangle_T \propto \left|\iint e^{ik(Yy+Zz)/R}\,dS\right|^2$$

or $$I(Y,Z) \propto \left(\iint e^{ik(Yy+Zz)/R}\,dS\right)\left(\iint e^{-ik(Yy+Zz)/R}\,dS\right)$$

Now it is clear that when $Y\to -Y$ and $Z\to -Z$ the irradiance remains the same.

ie, $I(-Y,-Z) = I(Y,Z)$. Note, we did not have to go to cylindrical coordinates and assume $\Phi = 0$

(10.25) The radius of the Airy disk (ie, 1st zero of $J_1(u)$) occurs when

$$ka \sin\theta = 3.83 \quad \text{(from Table 10.1)}$$

Also $\sin\theta = y/R$ where

y = diameter of spot on Moon
R = distance to Moon

ie $$\frac{ka\,y}{R} = 3.83$$

$$\frac{2\pi a y}{\lambda R} = 3.83$$

$$y = \frac{3.83\,\lambda R}{2\pi a}$$

$$\boxed{y = 2.90 \times 10^{5}\,m = 290\,km}$$

(10.28) $(\Delta\phi)_{min} = 1.22\ \lambda/D$. This is the equation for Rayleigh's criteron, namely when the max of one Airy disk falls at the min. of the second.

$$\Rightarrow (\Delta\phi)_{min} = 1.22\ \frac{550\ nm}{508\ cm} \times 10^{-7}\ \frac{cm}{nm}$$

(a) $\boxed{(\Delta\phi)_{min} = 1.32 \times 10^{-7}\ \text{radians}}$

(b) If looking at 2 objects separated by a distance d at a range R, they are just resolved when

$$(\Delta\phi)_{min} = d/R \Rightarrow d = R(\Delta\phi)_{min}$$

For Moon: $d = (3.844 \times 10^{8}\ m)(1.32 \times 10^{-7}\ rad.)$

$\boxed{d = 50.8\ m}$ using Mt Palomar telescope.

(c) For eye, $D = 4.00\ mm$

$$\Rightarrow d_{eye} = R_{earth-moon}\ \Delta\phi_{min-eye}$$

$$= R_{e-m}\ 1.22\ \frac{\lambda}{D_{eye}} = (3.844 \times 10^{8}\ m)\ 1.22 \left(\frac{550\ nm}{4.0\ mm}\right)$$

$\boxed{d = 64{,}500\ m}$ using eye.